

Math 221-Midterm 1 Practice exam 2

1. (20 points) Calculate the derivatives of the following functions by using differentiation rules.

(a) $y = x^5 - \frac{5}{\sqrt[5]{x}} + \frac{1}{3x^4}$

(b) $y = \frac{\sin x}{x^3 + 1}$

(c) $y = (x^3 + x + 5)^{12}$

(d) $y = \frac{x}{\sqrt{1 + \cos^2 x}}$

2. (9 points) Use **the definition of the derivative as a limit** to find the equation of the tangent line to the curve $y = \sqrt{3x^2 + 1}$ at $x = 1$.

3. (25 points) Compute the following limits or show that it does not exist. If you think the limit does not exist, explain why, and specify if the limit is ∞ or $-\infty$.

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^3 - 9x}$.

(b) $\lim_{x \rightarrow 2} \frac{\frac{1}{x^2} - \frac{1}{4}}{x - 2}$

(c) $\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 7x}$ (Hint: $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1.$)

(d) $\lim_{x \rightarrow 0} x^3 \cos\left(\frac{1}{x^2}\right)$

(e) $\lim_{x \rightarrow \infty} \frac{x^2 + 4x}{2x - 3}$

4. (10 points) For each of the problems, circle the (unique) correct answer.

(a) One of the following functions is continuous but not differentiable at $x = 0$. Which one?

A. $f(x) = x|x|$

B. $f(x) = \sin\left(\frac{1}{x}\right)$

C. $f(x) = x^{1/3}$

D. $f(x) = \tan x$

(b) $\lim_{x \rightarrow 4} \frac{x}{x^2 - 5x + 4}$ is

A. ∞

B. $-\infty$

C. 4.

D. None of the above.

(c) True or false? We know $\lim_{x \rightarrow 0} \frac{|x|}{x}$ does not exist because the bottom becomes 0 when $x = 0$.
 A. True B. False

(d) True or false? If $\lim_{x \rightarrow 0} f(x) = 2$, then $f(0) = 2$.
 A. True B. False

(e) True or false? There are functions whose graph crosses its asymptotes.
 A. True B. False

5. (18 points) Do the following problems about finding asymptotes.

(a) (8 points) Find all the horizontal asymptotes of $f(x) = \frac{x}{\sqrt{x^2 + 4}}$. Show your work.

(b) (10 points) Find *all* the asymptotes of $f(x) = \frac{x^2 + 2x}{x + 1}$. Show your work.

6. (18 points) Consider the function $f(x)$ defined by $f(x) = \begin{cases} (x+a)^2 & \text{for } x \leq 0 \\ 2x+b & \text{for } x > 0 \end{cases}$ and answer the following questions. Here a and b are parameters.

(a) State, in terms of limits, what it means for $f(x)$ to be continuous at $x = 0$.

(b) If $f(x)$ is continuous at $x = 0$, we should get a relation between a and b . What is that relation?

(c) For which values of a and b is $f(x)$ differentiable at $x = 0$? Justify your answer.